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# Finite element model to study calcium distribution in oocytes involving voltage gated $\text{Ca}^{2+}$ channel, ryanodine receptor and buffers



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## KEYWORDS

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**Abstract** Calcium is one of the most important signalling ions in cell biology performing numerous functions with high specificity. A calcium wave triggers life at fertilization but also can cause cell death. The means by which this single ion can be both highly specific and universal is believed to lie in its spatiotemporal dynamics mediated by ion channels, pumps, receptors and calcium buffers. During oocyte maturation the calcium signalling machinery undergoes differentiation which results in distinctly different calcium release patterns on all organizational scales from puffs to waves. The calcium concentration patterns required during different stages of oocyte maturation are still not completely known. Also the mechanisms involved in calcium dynamics in oocyte cell are still not well understood. In view of above a two dimensional model has been proposed to study calcium dynamics in an oocyte cell. The parameters such as buffers, ryanodine receptor and voltage gated calcium channel are incorporated in the model. Based on the biophysical conditions the initial and boundary conditions have been framed. The model is transformed into variational form and Ritz finite element method has been employed to obtain the solution. A program has been developed in MATLAB 7.10 for the entire problem and executed to obtain numerical results. The numerical results have been used to study the effect of buffers, RyR and VGCC on calcium distribution in oocyte. The results indicate that buffers can significantly decrease the calcium concentration and RyR & VGCC can significantly raise the calcium concentration level in the oocyte cell in order to initiate, sustain and terminate specific activities in the cell. The information generated from the model can be useful to biomedical scientists for clinical and biomedical applications.

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## 1. Introduction

$\text{Ca}^{2+}$  is a second messenger that mediates a plethora of cellular function ranging from neurotransmitter release to fertilization. Specially  $\text{Ca}^{2+}$  signalling is encoded in the spatial, temporal and amplitude features of cytoplasmic  $\text{Ca}^{2+}$  dynamics.<sup>1</sup> That

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is in the same cell  $\text{Ca}^{2+}$  signals of disparate duration, amplitude or frequency result in different cellular response. For example localized  $\text{Ca}^{2+}$  release through ryanodine receptor in vascular smooth muscle leads to vasodilation.<sup>2</sup> Whereas global sustained  $\text{Ca}^{2+}$  signals lead to vasoconstriction.<sup>3</sup>  $\text{Ca}^{2+}$  signals achieve this specificity by differentially activating  $\text{Ca}^{2+}$  dependent efforts based on their frequency, location, duration and amplitude. At fertilization, vertebrate eggs undergo a major transition from gametogenesis with dramatic cellular alteration referred to collectively as egg activation.  $\text{Ca}^{2+}$  is the universal signal for egg activation in all sexually reproducing species studied to date from plants to humans.<sup>4,6</sup> The fertilization induced  $\text{Ca}^{2+}$  signal has specific spatial and temporal dynamics which is essential to activate the egg and initiate embryonic development.<sup>4,5</sup> This specialized  $\text{Ca}^{2+}$  signal takes the form of a single or multiple  $\text{Ca}^{2+}$  transients depending on the species.<sup>4</sup> Changes in the concentration of cytosolic free calcium have been found to be responsible for the initiation and regulation of a variety of cellular functions including cellular proliferation, secretion, metabolic, adjustments and changes in gene expression.<sup>7,8</sup> The spatiotemporal patterns of  $[\text{Ca}^{2+}]_c$  as a result of agonist stimulation are as diverse as the roles of  $\text{Ca}^{2+}$  play in different cells. The temporal pattern of  $[\text{Ca}^{2+}]_c$  observed in a variety of cells includes oscillations or repetitive spiking.<sup>7,9–11</sup> Some cells, most notably *Xenopus* oocyte also exhibit interesting spatial patterns of  $[\text{Ca}^{2+}]_c$  including propagating waves and target and spiral patterns.<sup>12</sup>  $\text{Ca}^{2+}$  waves have also been observed in myocytes, astrocytes<sup>13</sup> hepatocytes<sup>10</sup> and airways epithelial cells.<sup>14</sup> The dynamics of  $\text{Ca}^{2+}$  is very important in cellular physiology because  $\text{Ca}^{2+}$  regulates their activity and interactions.<sup>15</sup>  $\text{Ca}^{2+}$  waves are dependent on the diffusion of  $\text{Ca}^{2+}$  ions both within and possibly between the cells: modulating  $\text{Ca}^{2+}$  ion diffusion may predictably alter the spatial and temporal character of the  $\text{Ca}^{2+}$  wave. Zeng and co-workers<sup>25</sup> developed a mathematical model of simulation of spontaneous  $\text{Ca}^{2+}$  oscillations in astrocytes mediated by voltage gated calcium channels (VGCC). A good number of attempts have been made by scientists on study of calcium distribution in neurons cells, astrocyte cells, but very few attempts are reported in the literature on modelling of calcium distribution in oocytes. No attempt is reported in the literature for modelling calcium distribution in oocytes in the presence of VGCC. In view of above a mathematical model has been developed to study effect of buffers and ryanodine receptor over  $\text{Ca}^{2+}$  profile in oocytes in the presence of VGCC. The model has been developed for a two dimensional unsteady state case. The finite element method<sup>17</sup> is employed to solve the problem. A computer program has been developed in MATLAB 7.10 for the whole problem and executed on Intel(R) Core™ i3 CPU, 4.00 GB RAM, 2.40 GHz processor.

## 2. Mathematical model and solution

Calcium kinetics in Oocytes is governed by a set of reaction–diffusion equations which can be framed assuming the following bimolecular reaction between  $\text{Ca}^{2+}$  and buffer species<sup>18,19</sup>



where  $[\text{Ca}^{2+}]$ ,  $[B_j]$  and  $[\text{CaB}_j]$  represent the cytosolic  $\text{Ca}^{2+}$  concentration, free buffer concentration and calcium bound buffer concentration respectively and 'j' is an index over buffer species,  $k_j^+$  and  $k_j^-$  are on and off rates for jth buffer respectively. Using Fickian diffusion, the buffer reaction diffusion system in two dimensions is expressed as<sup>20,30,32,33</sup>

$$\frac{\partial[\text{Ca}^{2+}]}{\partial t} = D_{\text{Ca}} \left( \frac{\partial^2[\text{Ca}^{2+}]}{\partial x^2} + \frac{\partial^2[\text{Ca}^{2+}]}{\partial y^2} \right) + \sum R_j + \sigma_{\text{VGCC}} + \sigma_{\text{RyR}} + \delta(x)\sigma_{\text{Ca}} \quad (2)$$

$$\frac{\partial[B_j]}{\partial t} = D_{B_j} \left( \frac{\partial^2[B_j]}{\partial x^2} + \frac{\partial^2[B_j]}{\partial y^2} \right) + \sum R_j \quad (3)$$

$$\frac{\partial[\text{CaB}_j]}{\partial t} = D_{\text{CaB}_j} \left( \frac{\partial^2[\text{CaB}_j]}{\partial x^2} + \frac{\partial^2[\text{CaB}_j]}{\partial y^2} \right) - \sum R_j \quad (4)$$

where reaction term  $R_j$  is given by

$$R_j = -k_j^+ [\text{Ca}^{2+}][B_j] + k_j^- [\text{CaB}_j] \quad (5)$$

$D_{\text{Ca}}$ ,  $D_{B_j}$ ,  $D_{\text{CaB}_j}$  are diffusion coefficients of free calcium, free buffer and  $\text{Ca}^{2+}$  bound buffer respectively. Where  $\sigma_{\text{VGCC}}$  is net influx of  $\text{Ca}^{2+}$  from the voltage gated calcium channel,  $\sigma_{\text{RyR}}$  is net influx of  $\text{Ca}^{2+}$  from the ryanodine receptor which is assumed to be within the cell i.e., at the centre ( $x = 2.5 \mu\text{m}$ ,  $y = 2.5 \mu\text{m}$ ) and  $\sigma_{\text{Ca}}$  is net influx of  $\text{Ca}^{2+}$  from the source and  $\delta(x)$  is the standard Dirac delta function placed at the  $\text{Ca}^{2+}$  source. Let  $[B_j]_T = ([B_j] + [\text{CaB}_j])$  be the total buffer concentration of jth buffer and the diffusion coefficient of buffer is not affected by the binding of calcium i.e.,  $D_{B_j} = D_{\text{CaB}_j}$ . Then Eq. (5) can be written as<sup>21</sup>

$$R_j = -k_j^+ [\text{Ca}^{2+}][B_j] + k_j^- ([B_j]_T - [B_j]) \quad (6)$$

It is assumed that the buffer concentration is present in excess inside the cytosol so that the concentration of free buffer is constant in space and time, i.e.  $[B_j] \equiv [B_j]_\infty$ . Under this assumption Eq. (3) is approximated by<sup>19</sup>

$$k_j^+ [\text{Ca}^{2+}][B_j] = k_j^- ([B_j]_T - [B_j]_\infty) \quad (7)$$

where  $[B_j]_\infty = \frac{k_j^- [B_j]_T}{(k_j^- + k_j^+ [\text{Ca}^{2+}]_\infty)}$  is the background buffer concentration. Thus for single mobile buffer species Eq. (2) can be written as<sup>18,19</sup>

$$\frac{\partial[\text{Ca}^{2+}]}{\partial t} = D_{\text{Ca}} \left( \frac{\partial^2[\text{Ca}^{2+}]}{\partial x^2} + \frac{\partial^2[\text{Ca}^{2+}]}{\partial y^2} \right) - k_j^+ [B_j]_\infty ([\text{Ca}^{2+}] - [\text{Ca}^{2+}]_\infty) + \sigma_{\text{VGCC}} + \sigma_{\text{RyR}} + \delta(x)\sigma_{\text{Ca}} \quad (8)$$

where  $D_{\text{Ca}}$  is the diffusion coefficient of free calcium,  $\delta(x)\sigma_{\text{Ca}}$  is the source amplitude due to the calcium channel.  $\sigma_{\text{VGCC}}$  is the flux due to VGCC and this has been modelled using the Goldman–Hodgkin–Katz (GHK) current equation.<sup>20,22</sup> We assume a single point source of  $\text{Ca}^{2+}$ ,  $\sigma_{\text{Ca}}$  at  $x = 0$ ,  $y = 0$ , there are no sources for buffers and buffer concentration is in equilibrium with  $\text{Ca}^{2+}$  far from the source and GHK equation as

$$I_{\text{Ca}} = P_{\text{Ca}} z_{\text{Ca}}^2 \frac{F^2 V_m}{RT} \frac{[\text{Ca}^{2+}]_i - [\text{Ca}^{2+}]_o \exp(-z_{\text{Ca}} \frac{FV_m}{RT})}{1 - \exp(-z_{\text{Ca}} \frac{FV_m}{RT})} \quad (9)$$

where  $[Ca^{2+}]_i$  and  $[Ca^{2+}]_o$  are the intracellular and extracellular calcium concentration respectively.  $P_{Ca}$  is the permeability of calcium ion,  $z_{Ca}$  is the valency of calcium ion.  $F$  is the Faradays constant.  $V_m$  is the membrane potential.  $R$  is gas constant and  $T$  is absolute temperature. Eq. (9) is converted into molar/s by using the following equation

$$\sigma_{VGCC} = \frac{-I_{Ca}}{z_{Ca} F V_{Oocyte}} \quad (10)$$

The negative sign in Eq. (10) is taken due to the inward current by convection. GHK current equation gives the current density as a function of voltage. The GHK equation is derived from the constant field which assumes that the electric field in the membrane is constant and thus ions move in the membrane in the same way as in free solution.  $\sigma_{RyR}$  is flux due to ryanodine receptor given as<sup>26,31</sup>

$$\sigma_{RyR} = V_{RyR} P_o ([Ca^{2+}]_{ER} - [Ca^{2+}]) \quad (11)$$

Combining Eqs. (8)–(11) we get proposed mathematical model as given below

$$\begin{aligned} \frac{\partial [Ca^{2+}]}{\partial t} = & D_{Ca} \left( \frac{\partial^2 [Ca^{2+}]}{\partial x^2} + \frac{\partial^2 [Ca^{2+}]}{\partial y^2} \right) \\ & - k_j^+ [B_j]_{\infty} ([Ca^{2+}] - [Ca^{2+}]_{\infty}) + P_{Ca} z_{Ca}^2 \frac{F^2 V_m}{RT} \\ & \times \frac{[Ca^{2+}]_i - [Ca^{2+}]_o \exp(-z_{Ca} \frac{F V_m}{RT})}{1 - \exp(-z_{Ca} \frac{F V_m}{RT})} \\ & + V_{RyR} P_o ([Ca^{2+}]_{ER} - [Ca^{2+}]) + \delta(x) \sigma_{Ca} \end{aligned} \quad (12)$$

The point source of calcium is assumed at  $x = 0, y = 0$  and as we move away from the source, the calcium concentration achieves its background value i.e.,  $0.1 \mu M$ . Thus the initial and boundary conditions for the above problem are<sup>30,32,33</sup>

Initial Condition:

$$[Ca^{2+}]_{t=0} = 0.1 \mu M \quad \forall \quad x, y \quad (13)$$

Boundary Conditions:

$$\lim_{x \rightarrow 0, y \rightarrow 0} -D_{Ca} \frac{\partial [Ca^{2+}]}{\partial x} = \sigma_{Ca} \quad (14)$$

$$\lim_{x \rightarrow 5, y \rightarrow 5} [Ca^{2+}] = 0.1 \mu M \quad (15)$$

Here  $[Ca^{2+}]$  is the background calcium concentration,  $P_{Ca} [Ca^{2+}]$  represents the rate of calcium efflux from the cytosol into extracellular space.  $\sigma_{Ca}$  represents the flux due to  $[Ca^{2+}]$  and incorporated on the boundary tends to the background concentration of  $0.1 \mu M$  as  $x \rightarrow \infty, y \rightarrow \infty$  but the domain taken by us is not infinite one. Here we are taking the distance required for  $[Ca^{2+}]$  to attain background concentration as  $5 \mu m$  for Oocyte along  $x$ -axis and  $y$ -axis.<sup>30,32,33</sup> Our problem is to solve Eq. (12) coupled with Eqs. (13)–(15). For our convenience we are writing ‘ $u$ ’ in lieu of  $[Ca^{2+}]$ . From (12) we get

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \alpha u + \beta - \frac{1}{D_{Ca}} \frac{\partial u}{\partial t} = 0 \quad (16)$$

$$\begin{aligned} \alpha = & \frac{1}{D_{Ca}} \left[ k_j^+ [B_j]_{\infty} - P_{Ca} z_{Ca} \frac{\frac{F^2 V_m}{RT}}{1 - \exp(-z_{Ca} \frac{F V_m}{RT})} + V_{RyR} P_o \right], \beta \\ = & \frac{1}{D_{Ca}} \left[ k_j^+ [B_j]_{\infty} u_{\infty} - \frac{P_{Ca} z_{Ca} \frac{F^2 V_m}{RT} \exp(-z_{Ca} \frac{F V_m}{RT})}{1 - \exp(-z_{Ca} \frac{F V_m}{RT})} u_o + V_{RyR} P_o u_{ER} \right] \end{aligned} \quad (17)$$

Applying finite element method on Eq. (16) we can get variational form as

$$\begin{aligned} I^{(e)} = & \frac{1}{2} \int \int_A \left[ \left( \frac{\partial u^{(e)}}{\partial x} \right)^2 + \left( \frac{\partial u^{(e)}}{\partial y} \right)^2 + \alpha u^{(e)2} - 2\beta u^{(e)} + \frac{1}{D_{Ca}} \left( \frac{\partial u^{(e)}}{\partial t} \right)^2 \right] dA \\ & - \mu^{(e)} \int_{\partial A} \left( \frac{\sigma}{2D_{Ca}} u^{(e)} \right)_{x=5} dy \end{aligned} \quad (18)$$

Here we have used ‘ $u$ ’ in lieu of  $[Ca^{2+}]$  for our convenience and  $e = 1, 2, 3, \dots, 162$ . Also  $\mu^{(e)} = 1$  for  $e = 1$  and  $\mu^{(e)} = 0$  for rest of elements. The following linear shape function for calcium concentration within each element has been taken as<sup>17</sup>

$$u^{(e)} = c_1^{(e)} + c_2^{(e)} x + c_3^{(e)} y \quad (19)$$

$$u^{(e)} = p^T c^{(e)} \quad (20)$$

where

$$p^T = [1 \quad x \quad y] \quad (21)$$

and

$$c^{(e)T} = [c_1^{(e)} \quad c_2^{(e)} \quad c_3^{(e)}] \quad (22)$$

Substituting nodal conditions in Eq. (21), we get

$$\bar{u}^{(e)} = P^{(e)} * c^{(e)} \quad (23)$$

where

$$\bar{u}^{(e)} = \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \quad \text{and} \quad P^{(e)} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \quad (24)$$

From the Eq. (23), we have

$$c^{(e)} = R^{(e)} * \bar{u}^{(e)} \quad (25)$$

where

$$R^{(e)} = P^{(e)-1} \quad (26)$$

Substituting  $c^{(e)}$  from Eq. (25) in (20), we get

$$u^{(e)} = p^T R^{(e)} \bar{u}^{(e)} \quad (27)$$

The expression (18) can be written as

$$I^{(e)} = I_l^{(e)} + I_m^{(e)} - I_n^{(e)} - I_k^{(e)} + I_p^{(e)} \quad (28)$$

where

$$I_l^{(e)} = \frac{1}{2} \int \int_A \left[ \left( \frac{\partial u^{(e)}}{\partial x} \right)^2 + \left( \frac{\partial u^{(e)}}{\partial y} \right)^2 \right] dA \quad (29)$$

$$I_m^{(e)} = \frac{1}{2} \alpha \int \int_A u^{(e)2} dA \quad (30)$$

$$I_n^{(e)} = 2\beta \int \int_A u^{(e)} u_{\infty} dA \quad (31)$$

$$I_k^{(e)} = \frac{\mu^{(e)}}{2} \int_{\partial A} \left[ \frac{\sigma_{Ca}}{D_{Ca}} u^{(e)} \right]_{x=5} dy \quad (32)$$

$$I_p^{(e)} = \frac{1}{2D_{Ca}} \frac{d}{dt} \int_A (u^{(e)})^2 dA \quad (33)$$

Now we extremize the integral  $I^{(e)}$  w.r.t. each nodal calcium concentration  $u_i$  as given below

$$\frac{dI}{d\bar{u}} = \sum_{e=1}^{162} \bar{M}^{(e)} \frac{dI^{(e)}}{d\bar{u}^{(e)}} \bar{M}^{(e)^T} = 0 \quad (34)$$

and

$$\frac{dI^{(e)}}{d\bar{u}^{(e)}} = \frac{dI_l^{(e)}}{d\bar{u}^{(e)}} + \frac{dI_m^{(e)}}{d\bar{u}^{(e)}} - \frac{dI_n^{(e)}}{d\bar{u}^{(e)}} - \frac{dI_k^{(e)}}{d\bar{u}^{(e)}} + \frac{d}{dt} \frac{dI_p^{(e)}}{d\bar{u}^{(e)}} \quad (35)$$

where

$$\bar{M}^{(e)} = \begin{bmatrix} 0 & 0 & 0 \\ \cdot & \cdot & \cdot \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ \cdot \\ u_{100} \end{bmatrix} \quad (36)$$

$$\text{and} \quad I = \sum_{e=1}^{162} I^{(e)} \quad (37)$$

This leads to a following system of linear differential equations.

$$[A]_{100 \times 100} \left[ \frac{\partial \bar{u}}{\partial t} \right]_{100 \times 1} + [B]_{100 \times 100} [\bar{u}]_{100 \times 1} = [C]_{100 \times 1} \quad (38)$$

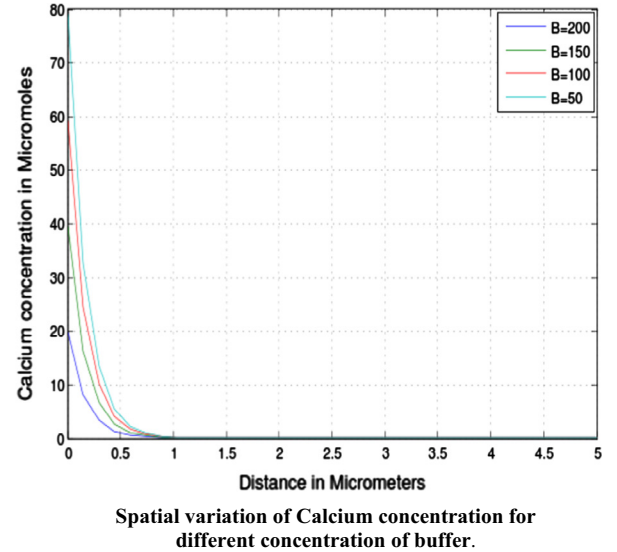
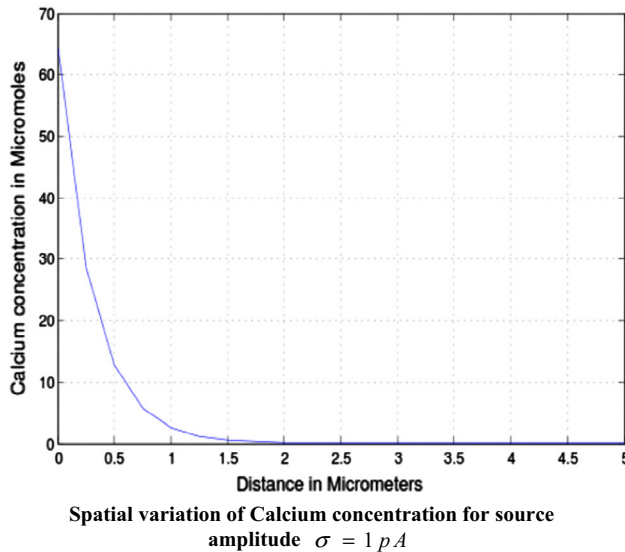
Here  $[\bar{u}] = [u_1, u_2, u_3 \dots u_{100}]$   $A$  and  $B$  are the system matrices and  $C$  is the system vector. The Crank–Nicolson method is used to solve the system of differential Eq. (38). A computer program has been developed in MATLAB 7.10 for the entire problem and executed on Intel(R) Core™ i3 CPU, 4.00 GB RAM, 2.40 GHz processor. The numerical values of biophysical parameters used in the model are stated in Table 1.<sup>16,23,24,31</sup>

### 3. Results and discussion

In this part, we have shown the numerical results for calcium profile against different biophysical parameters which have been obtained using values of parameters given in Table 1 unless stated along with figures.

From Fig. 1 the effect of source amplitude and the buffers is clearly visible. We see that the calcium concentration is higher near the source and decreases slowly as we move away from the source. The concentration is higher from 0  $\mu\text{m}$  to 1  $\mu\text{m}$  and then decreases slowly up to 1.5  $\mu\text{m}$  and finally tends to its initial value of 0.1  $\mu\text{M}$  for the source amplitude. The figure

Symbol	Parameter	Value
$D_{Ca}$	Diffusion coefficient	250 $\mu\text{m}^2/\text{s}$
$k_j^+$	On rate for EGTA	3/ $\mu\text{M s}$
$k_j^-$	Off rate for EGTA	1/s
$k_j^+$	On rate for BAPTA	100/ $\mu\text{M s}$
$k_j^-$	Off rate for BAPTA	10/s
$[B]_\infty$	Total buffer concentration	50 $\mu\text{M}$
$\sigma$	Source amplitude	1 pA
$[Ca^{2+}]_o$	Extracellular Calcium Concentration	3 mM
$P_{Ca}$	Calcium permeability	$4.3 \times 10^{-8}$ m/s
$V_m$	Membrane potential	-0.05 V
$z_{Ca}$	Valency of calcium	2
$V_{Oocyte}$	Volume of oocyte cytosol	$5.48 \times 10^{-11}$ l
$F$	Faraday's constant	96487 C/mole
$R$	Gas constant	8.314 J/K mole
$T$	Absolute temperature	37 °C
$P_o$	Rate of calcium efflux	0.5 M/s
$V_{RyR}$	RyR receptor rate	0.5 $\mu\text{M/s}$
$[Ca^{2+}]_{ER}$	ER $Ca^{2+}$ concentration	500 $\mu\text{M}$



**Figure 1** Spatial variation of calcium concentration for source amplitude  $\sigma = 1$  pA and for different concentration of buffer.

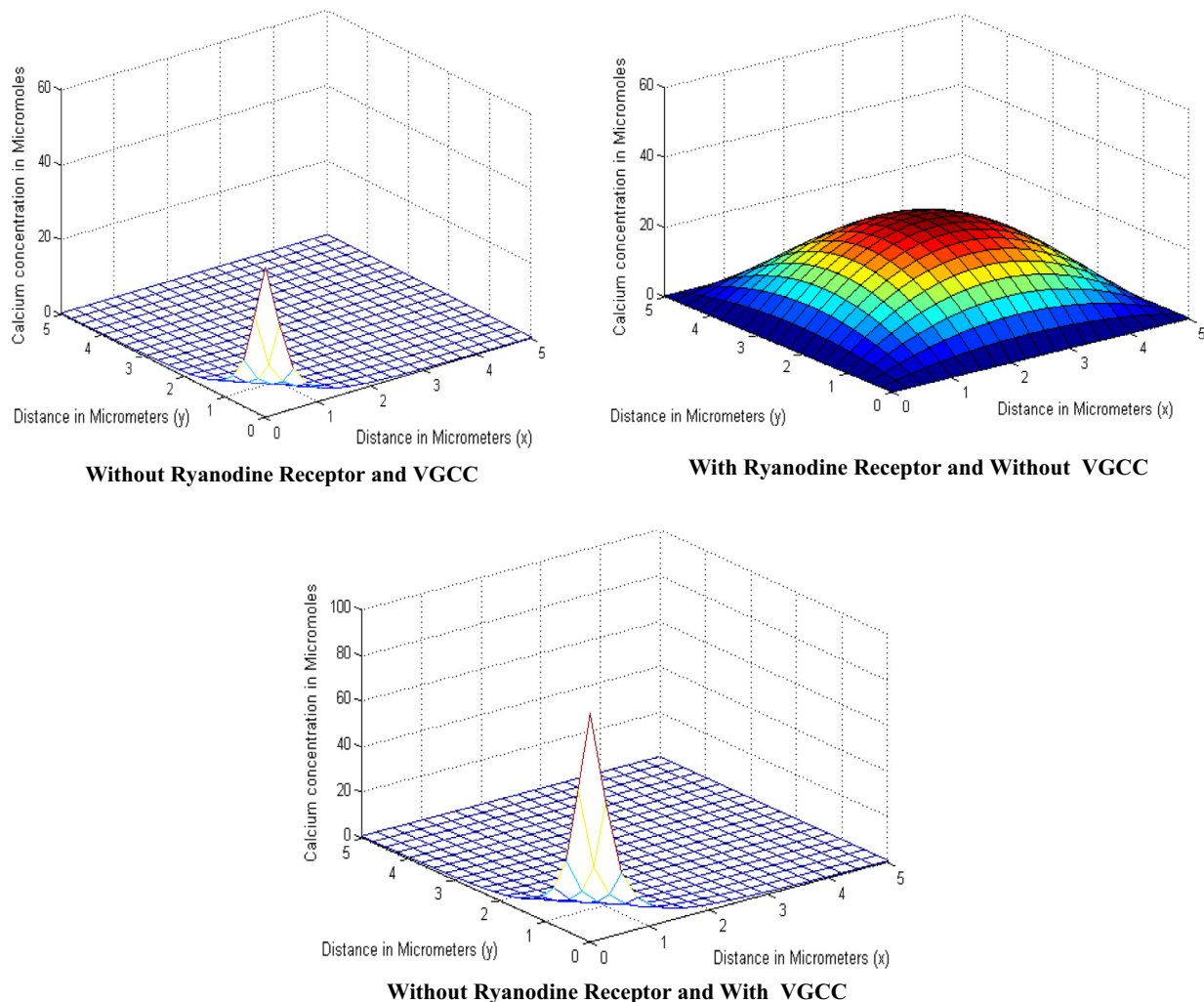
further shows that the calcium concentration is higher for lower value of buffer this is because the higher concentration of buffer binds much calcium and makes the system to attain the steady state earlier. The calcium concentration is higher from  $0\ \mu\text{M}$  to  $0.5\ \mu\text{M}$  and then onwards decreases gradually to attain the initial value.

Fig. 2 shows the effect of parameters i.e., the calcium concentration with and without these parameters. The figure shows that the calcium concentration is lower in the absence of parameters and very much higher in the presence of them. This is clearly visible from the figure because without these parameters there is less calcium to diffuse as these parameters contribute nothing because of their absence, but the calcium concentration is higher in the presence of these parameters as they release the calcium and contribute in the diffusion so as to make the concentration high. The figure shows the  $\text{Ca}^{2+}$  concentration is low in the absence of VGCC & RyR which is  $15\ \mu\text{M}$  and increases when RyR is involved and reaches up to  $25\ \mu\text{M}$ . Further the  $\text{Ca}^{2+}$  concentration reaches the higher level of  $40\ \mu\text{M}$  when VGCC is present.

Fig. 3 gives the spatial variation of calcium concentration for different time periods. We observe that at

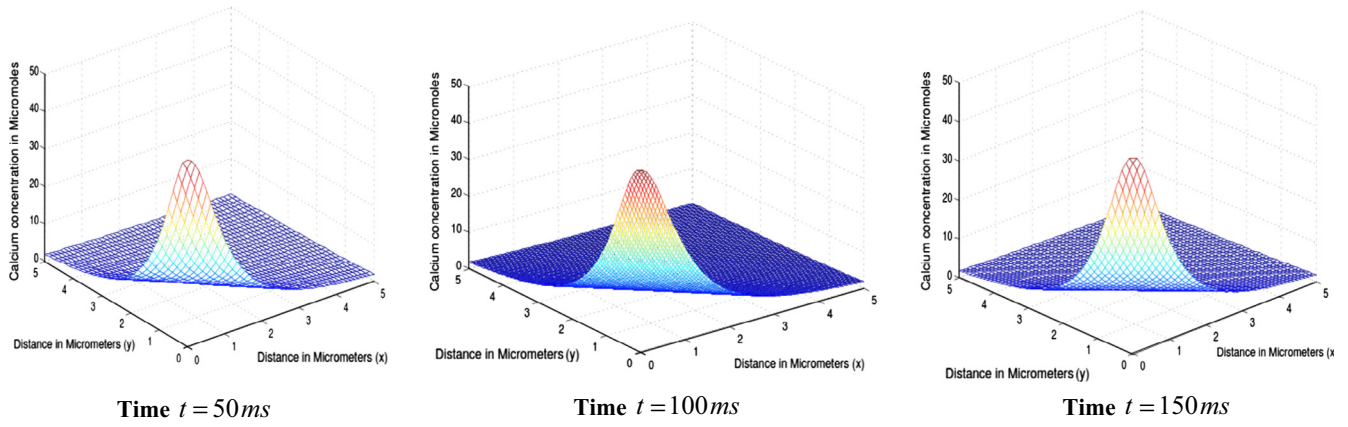
$t = 50\ \text{ms}$ ,  $100\ \text{ms}$ ,  $150\ \text{ms}$  the peak calcium concentration is  $27\ \mu\text{M}$ ,  $28\ \mu\text{M}$ ,  $30\ \mu\text{M}$  respectively. We observe that the peak  $\text{Ca}^{2+}$  concentration at source increases with time. Also the spread of  $\text{Ca}^{2+}$  increases with time.

Fig. 4 shows the steady state effect of increasing buffer concentration of EGTA buffer on calcium distribution in oocytes for source amplitude  $1\ \text{pA}$ . In all the four cases different equilibrium concentrations ( $[B]_{\infty}$ ) of EGTA buffer is taken to be  $B = 50, 100, 150, 200\ \mu\text{M}$  respectively. From the figure it is evident that as buffer concentration increases, the peak cytosolic calcium concentration at source decreases from  $30\ \mu\text{M}$  for  $B = 50\ \mu\text{M}$  to  $10\ \mu\text{M}$  for  $B = 200\ \mu\text{M}$ . Our results suggest that if buffer concentration increases too much then it either balkanizes  $\text{Ca}^{2+}$  signal or makes it less effective. In our calculations we also observed that buffer concentration alters the time required to achieve the steady state. Here we further observe that as buffer concentration increases, the area of dispersion of cytosolic calcium concentration decreases. The results obtained in this study are in a close agreement with the experimental studies<sup>27,28</sup> and the results obtained by Panday and Pardasani.<sup>29</sup> and Jha et al.<sup>30</sup>.

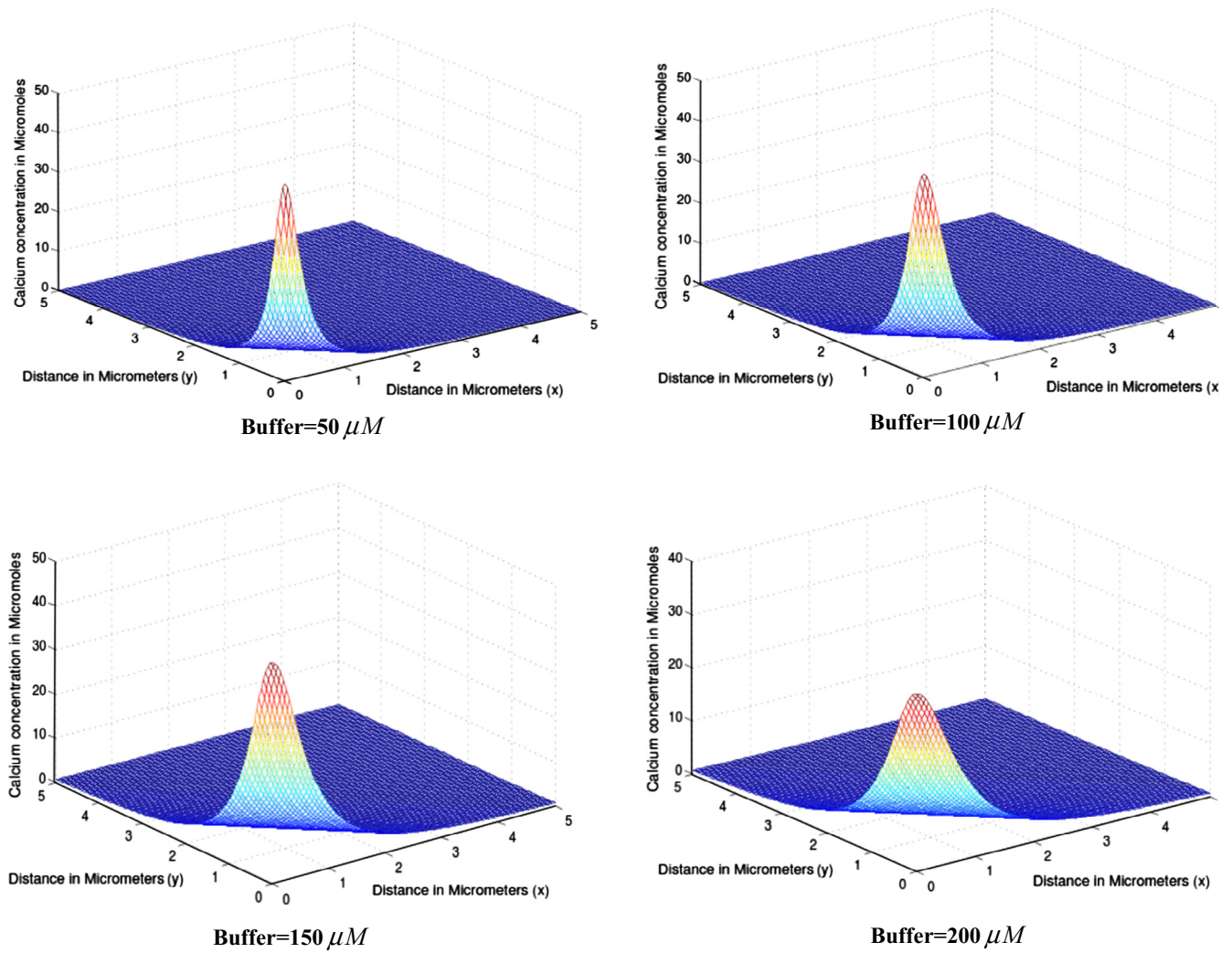


**Figure 2** Calcium concentration in the absence and the presence of RyR and VGCC.





**Figure 3** Spatial variation of calcium concentration for different time periods.



**Figure 4** Spatial variation of calcium concentration for different concentrations of EGTA buffer.

#### 4. Conclusion

The two-dimensional finite element model has been proposed for an unsteady state case and employed to study effect of VGCC, RyR and buffers on calcium concentration

distribution in oocytes. The results provide us information about spatio-temporal relationship of calcium concentration with buffers, RyR and VGCC in oocyte cell. It is concluded from results that VGCC and RyR can significantly raise the level of calcium concentration in the cell in response to the

requirements for initiation, sustenance of the specific activities of the cell. Also the buffers are capable of lowering down the calcium concentration in the cell when the calcium concentration becomes high in the cell for a specific activity. The cell exhibits a beautiful coordination of RyR, VGCC and buffers in order to regulate the calcium concentration in the oocyte cell. The finite element method is quite flexible and versatile in the present situation as it has been possible to incorporate the important parameters such as VGCC, RyR and buffers in the model. Such models can be developed further to study the relationship among various parameters under normal and abnormal conditions to generate the information which can be of great use to biomedical scientists for developing protocols for diagnosis and treatment of diseases related to reproduction.

### Conflict of interest

The authors declare that there is no any kind of conflict of interest.

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